CYCLIC LOADING OF RUBBERS - AMPLITUDE SPECTRUM AND PAYNE EFFECT

Kliment Hadjov, Verjinia Aleksandrova

Department of Applied Mechanics
University of Chemical Technology and Metallurgy
8 Kl. Ohridski, 1756 Sofia, Bulgaria
E-mail: klm@uctm.edu

Received 20 July 2016
Accepted 16 December 2016

ABSTRACT

Introducing nonlinear integral constitutive equation with singular kernels the authors have obtained the stress-strain hysteresis curves in the case of imposed large strains (stresses), respectively. The respective solutions of the integral equations are represented in Fourier series, which coefficients being convergent are used to obtain the Fourier amplitude spectrum and the amplitude dependence (decreasing) of the storage modulus - the so called Payne effect. The respective enhancement of the compliance with the imposed stress amplitude is also discussed. The experimental data obtained for polyisoprene rubber well agree with the theoretical predictions.

Keywords: singular kernels, stress (strain) responses, Mullins, Payne effects, damping function, Ogden equation.

INTRODUCTION

The more important result related to the self-heating and the vibration attenuation capability of rubbers is the hysteresis loop. In the general nonlinear viscoelastic case this loop can be obtained from the solution of the constitutive mechanical stress-strain equation [1 - 5].

In [3] introducing a nonlinear constitutive integral equation with singular kernel is obtained the hysteresis loop curves in the case of imposed strains and stresses, respectively. The Mullins effect is taken into account using a damping function related with the initial damage by large strains. The nonlinearity due to these strains is taken into account using the Ogden equation. The Integral equation is as follows:

$$\sigma(t) = \chi(\varepsilon(t)) - \int_0^t R(t-\tau)\chi(\varepsilon(\tau))d\tau \quad \text{with}$$

$$\chi(\varepsilon(\mathrm{t})) = \varphi(\varepsilon_{imp}(t))\mathrm{g}(\varepsilon_{imp}(\mathrm{t})) \qquad (1)$$

$$\varphi(\varepsilon) = \sum_{i=1}^{3} \mu_i (\lambda(\varepsilon)^{\kappa_i - 1} - \lambda(\varepsilon)^{-\frac{\kappa_i}{2} - 1})$$

$$R(t) = \sum_{i=1}^{3} A_i \frac{e^{-\beta_i t}}{t^{\alpha_i}}, \quad g(t) = 1 - C\frac{1}{\frac{1}{\varepsilon_{imp}(t)} + 1}$$

Here $\sigma(t)$ is the stress as a function of time t; $\varepsilon(t)$ is the imposed strain; $R(t-\tau)$ is the relaxation kernel; $\varphi(\varepsilon(t)$ is the instantaneous stress-strain curve (Ogden equation in the uniaxial case); $g(t)$ is the damping function with his damage parameter C; $\mu_1, \mu_2, \mu_3, \kappa_1, \kappa_2, \kappa_3$ are parameters obtained from instantaneous stress-strain test and the stretch λ is related with the engineering strain as follows $\lambda(\varepsilon) = 1 + \varepsilon$. Equation (1) represents the stress response in the case of imposed strain law. The solution of equation (1) can be represented as follows [3]:

$$\varepsilon(t) = \chi^{-1}(y(t) \qquad \text{with}$$

$$y(t) = \sigma_{imp}(t) + \int_0^t K(t-\tau)\sigma_{imp}(\tau)d\tau \qquad (2)$$

Here $\varepsilon(t) = \chi^{-1}(y(t)$ is the inverse function of $\chi(y)$ and $K(t-\tau)$ is the creep kernel which has the form

$$K(t) = \frac{e^{-\beta_i t}}{t} \sum_{n=1}^{\infty} A_i \Gamma(\alpha_i)^n t^{\alpha_i n} / \Gamma(\alpha_i n)$$

$\Gamma(\alpha)$ is the gamma function [1 - 4]. Equation (2) represents the strain response in the case of imposed stresses.

General framework

The Payne effect is an amplitude dependent softening phenomenon. It is characterized by a decrease in the storage modulus with increasing strain amplitude by cycling loading [5]. In order to obtain the relation between storage modulus and the strain amplitude in the case of large strains are employed the Fourier series decomposition of the stress response obtained as a solution of the nonlinear integral hereditary equation.

The damping function in the case of imposed strains and stresses via the time can be represented as [3]:

$$g(t,\varepsilon_o) = 1 - C(\varepsilon_o)\frac{1}{\dfrac{1}{\varepsilon_{imp}(t,\varepsilon_o)} + 1}$$

$$g(t,\sigma_o) = 1 + C(\sigma_o)\frac{1}{\dfrac{1}{\sigma_{imp}(t,\sigma_o)} + 1} \qquad (3)$$

The first equation (3) is illustrated on Fig. 1. To retain the damping function constant after the first cycle here is introduced the Heaviside function.

The damping function is parameter C dependent, which on the other hand is strain amplitude dependent. Thus the damping function is related with the strain amplitude and this relation should be incorporated in the respective stress response via the nonlinearity function (see equation (1)). Finally a relation between the storage modulus and the imposed strain amplitude is obtained.

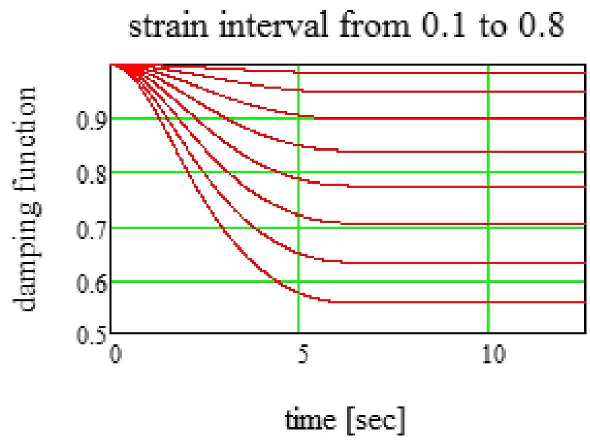

Fig. 1. Damping function via time for different strain amplitudes.

The Payne effect is checked using this way. In order to achieve this, the respective responses in Fourier series must be represented and after summation of the first three members is obtained good approximation of the responses.

Fourier amplitude spectrum

This spectrum represents a relation between imposed amplitude and frequency [6]. In the case of imposed strains the Fourier expansion is (here is introduced equation (1)):

$$\sigma(t)/\varepsilon_o = E''_o + \sum_{n=1}^{\infty} E''_n \cos n\omega t + E'_n \sin n\omega t \qquad (4)$$

with $E''_o = \dfrac{1}{T}\displaystyle\int_0^T \sigma(t)dt$,

$$E''_n = \frac{2}{T}\int_0^T \sigma(t)\cos n\omega t dt$$

and $E'_n = \dfrac{2}{T}\displaystyle\int_0^T \sigma(t)\sin n\omega t dt \qquad (5)$

In the case of imposed stresses the Fourier expansion, introducing here the solution (2), is

$$\varepsilon(t)/\sigma_o = S''_o + \sum_{n=1}^{\infty} S''_n \cos n\omega t + S'_n \sin n\omega t \qquad (6)$$

with $S''_o = \frac{1}{T}\int_0^T \varepsilon(t)dt$, $\quad S''_n = \frac{2}{T}\int_0^T \varepsilon(t)\cos n\omega t dt$

and $S'_n = \frac{2}{T}\int_0^T \varepsilon(t)\sin n\omega t dt$ (7)

The relation between the strain (stress) amplitude and the respective Fourier coefficients as a function of the imposed frequency can be obtained from [6]:

$$\varepsilon_0(n,\omega) = \sqrt{(a_n(n,\omega))^2 + (b_n(n,\omega))^2} \quad (8)$$

here $E''_o = a_0$, $E''_n = a_n$, $E'_n = b_n$

or $S''_o = a_0$, $S''_n = a_n$, $S'_n = b_n$ respectively.

The Payne effect

Concerning the Payne effect must be obtained a relation between the storage modulus and the imposed strain amplitude. To do this in the case of imposed large strains is used the third equation (5) and after summation of the first three members is arrived to the desired relation. Thus, supposing convergence of the Fourier coefficients the storage modulus can be defined as:

$$E'(\varepsilon_o) = \sum_{n=1}^{\infty} E'_n(\varepsilon_o) \quad (9)$$

In the case of imposed large stresses is used the third equation (7) and after summation of the first two members the desired relation is obtained and the storage compliance is defined as:

$$S'(\sigma_o) = \sum_{n=1}^{\infty} S'_n(\sigma_o) \quad (10)$$

To obtain the approximation error in the first case (imposed strains) in % by stopping at the third expansion member is used the following relative difference:

$$\text{Error}(\varepsilon_o) = \frac{E'4(\varepsilon_o)\text{-}E'3(\varepsilon_o)}{E'4(\varepsilon_o)}.100 \quad (11)$$

To obtain the approximation error in the second case (imposed stresses) in % by stopping at the second expansion member, we use the following relative difference:

$$\text{Error}(\sigma_o) = \frac{S'3(\sigma_o)\text{-}S'2(\sigma_o)}{S'3(\sigma_o)}.100 \quad (12)$$

The convergence in this case is guaranteed. In [6] authors have proved that the sums

$\sum_{n=1}^{\infty}\sqrt{(a_n)^2 + (b_n)^2}$ and

$\sum_{n=1}^{\infty}|a_n| + |b_n|$ converge and in the case of positive a_n, b_n

the sum $\sum_{n=1}^{\infty} a_n + b_n$ converges too. Here a_n, b_n are the Fourier coefficients related with the storage or compliance modulus according to definitions (13) and (15), respectively. The fact that an integral hereditary equation can take into account the Payne effect is discussed in [5].

RESULTS AND DISCUSSION

In this work a polyisoprene rubber is used [3, 4], produced in University of Chemical Technology and Metallurgy, Sofia. The kernel parameters are respectively:

$A_1 = 0.031, A_2 = 0.001, A_3 = 0.007,$
$\alpha_1 = 0.19, \alpha_2 = 0.77, \alpha_3 = 0.43,$
$\beta_1 = 0.0062, \beta_2 = 0.08, \beta_3 = 0.039$

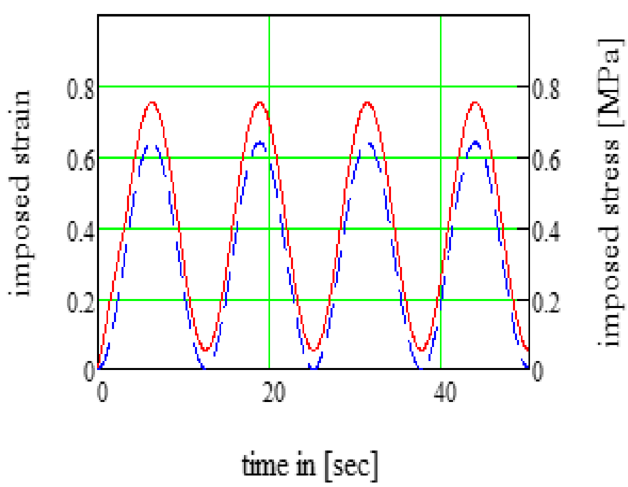

Fig. 2. Imposed strains and stresses as a function of time, s.

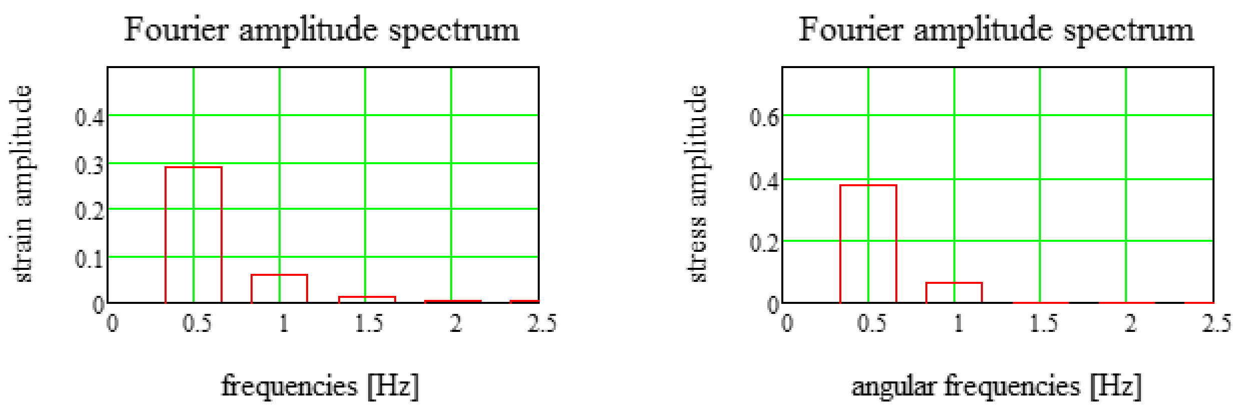

Fig. 3. Fourier amplitude spectrum. Left imposed strains, right-imposed stresses, MPa.

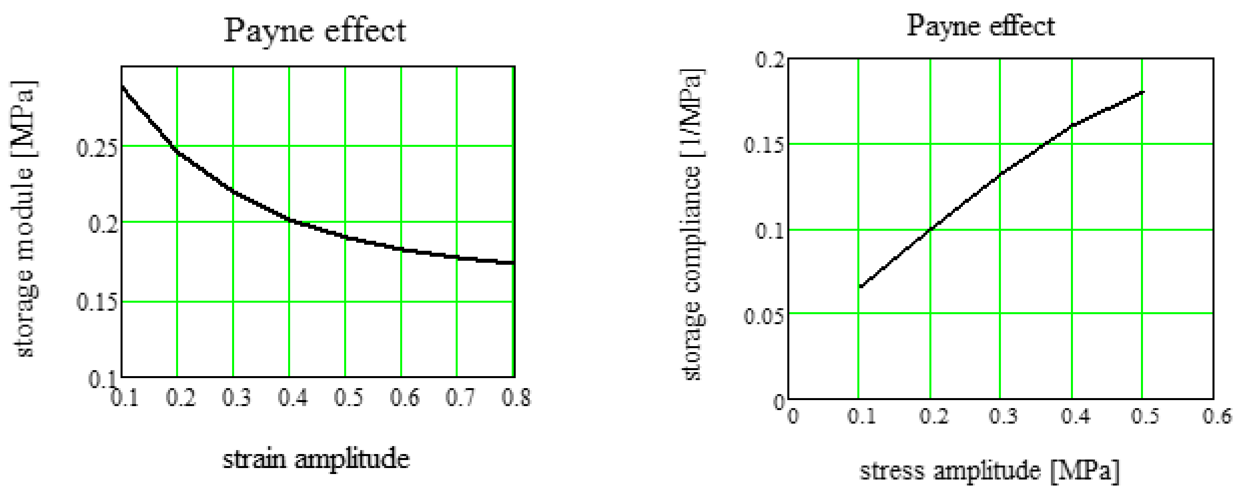

Fig. 4. Polyisoprene rubber. Payne effect. Left-storage modulus via strain amplitude. Right-storage compliance via stress amplitude.

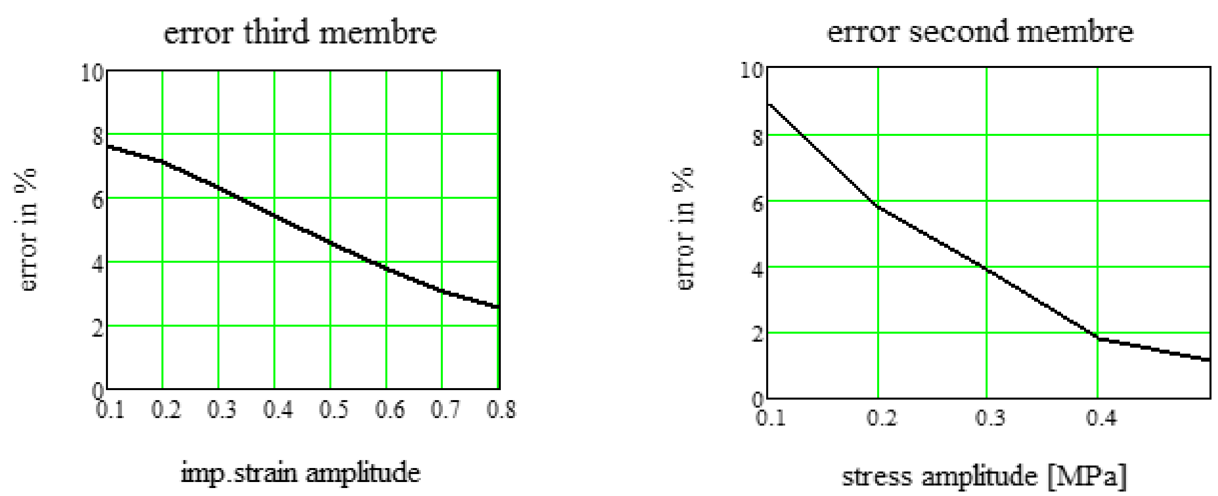

Fig. 5. Errors via the imposed strain (stress) amplitude in %.

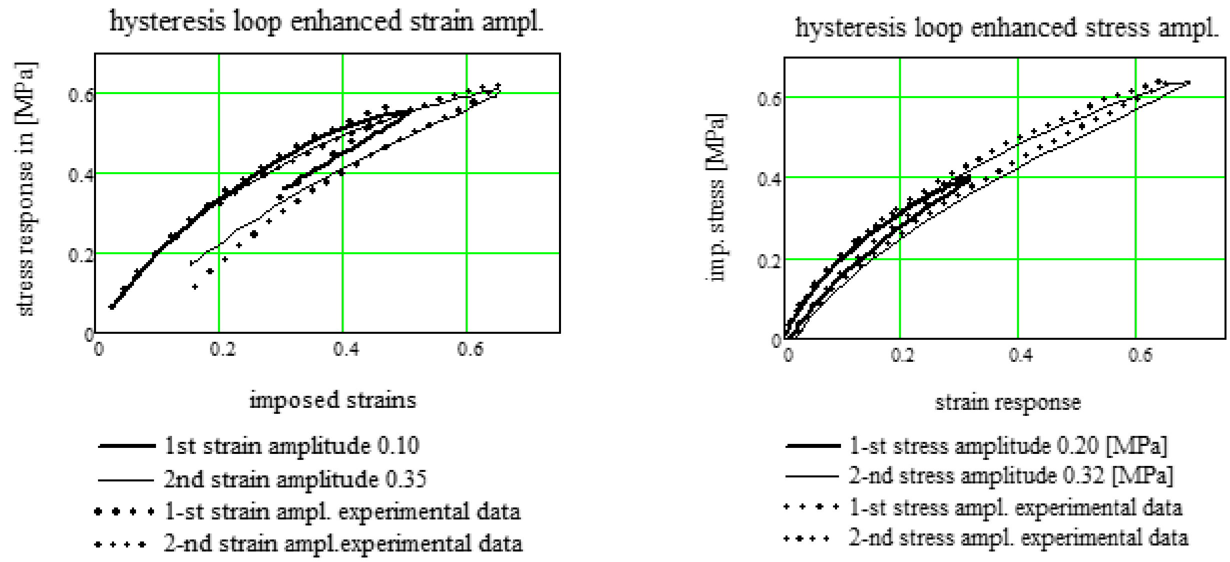

Fig. 6. Polyisoprene rubber. Hysteresis loops. Left-two imposed strain amplitudes $\varepsilon_{0,1} = 0.1$, $\varepsilon_{0,2} = 0.35$. Right - two imposed stress amplitudes $\sigma_{0,1} = 0.2$, $\sigma_{0,2} = 0.32$, MPa. Points - experimental data.

The Ogden parameters are as follows:

$$\mu_1 = 0.13, \mu_2 = -1.46, \mu_3 = 6.2x10^{-3},$$
$$\kappa_1 = 2.35, \kappa_2 = -0.991, \kappa_3 = 5.74$$

The imposed strain and stress laws are illustrated on Fig. 2.

The parameters, characterizing these laws, were: strain amplitude $\varepsilon_0 = 0.35$, stress amplitude $\sigma_0 = 0.32$, frequency $\omega = 0.5$.

On Fig. 3 is illustrated the Fourier amplitude spectrums according to equations (8) and the importance of each member and thus confirm equations (11) and (12). On Fig. 4 is illustrated the Payne softening effect (storage modulus diminution and compliance enhancement with imposed impact amplitude) according to equations (9) and (10), respectively.

On Fig. 6 is illustrated the hysteresis loops for polyisoprene rubber according to equations (8, 10) and the respective experimental curves, obtained with the device described in [4]. Here the Payne effect can be seen too.

CONCLUSIONS

Using nonlinear integral equations with damping function whose single parameter can be obtained from independent experimentation are obtained the stress (strain) responses in the case of imposed strains (stresses) taking into account the Mullins and Payne softening effects. The experimental hysteresis curves agree with the theoretical ones obtained from the stress (strain) responses by imposing sinusoidal pulsations to the strain (stress) laws. It can be concluded that in the case of imposed strains the hysteresis loop area (related with the heat losses) is more pronounced as in the case of imposed stresses. The respective errors stopping to the third and second terms in the Fourier series in the case of large imposed impacts do not exceed 6 % and 7 %, respectively.

Acknowledgements

The work is supported by the Research Department of University of Chemical Technology and Metallurgy, Sofia, contract number 11562/2016.

REFERENCES

1. K. Hadjov, D. Dontchev, T. Hrima, V. Aleksandrova, On The Integral Equations of Volterra Concerning the Large Time Scale, International Journal of Engineering, t. XII, Fascicule 2, 2014.

2. T. Hrima et al., Loss Factor of Rubbers as a Function of Strain Amplitude and Frequency, Journal of the Bolkan Tribological Association, 2016, (in press).
3. T. Hrima, K. Hadjov, D. Dontchev, Stress and Strain Controlled Hysteresis of Rubbers, International Journal of Materials Engineering, 6, 2, 2016, 47-50.
4. K. Hadjov, Nonlinear elastoviscosity of rubbers by cycling loading, J .Chem. Technol. Metall., 48, 6, 2013.
5. O. Fang et al., Investigation of dynamic characteristics of nano-size calcium carbonate added in natural rubber vulcanizate, Composites: Part B, 60, 2014, 561-567.
6. A. Das, Signal Conditioning, Signals and Communication Technology, DOI: 10.1007/978-3-642-28818-02, Springer-Verlag Berlin, Heidelberg, 2012.