A TWO-ZONE MODEL OF BROWDENING DURING ROLLING

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ABSTRACT

The paper presents a mathematical model of the process of broadening during rolling of strips and sheets. A feature of the model is the consideration of the metal flow in two zones of plastic deformation hearth: advancing zone and lag zone. The mathematical model is based on the variational principle of minimum total capacity of rolling (the principle of Jourdain). In accordance with this principle, the metal flow velocities in the plastic deformation hearth should be varying. The velocity of the metal transversal flow in the hearth of deformation is represented as a power function of the transversal coordinate. Three parameters are to be varied: value of total broadening, value of broadening in the lag zone and degree of the dependence of transversal metal flow velocity. These parameters are determined by the Ritz method. Comparison of the results of experiments and the mathematical model showed adequacy.

Keywords: broadening during rolling, advance zone, lag zone, variation principle of Jourdain, Ritz method.

INTRODUCTION

In order to implement plans to stimulate the economy of Kazakhstan and Russia in the global crisis in various industries, such as machinery, metallurgy, construction, etc. it is necessary to provide these industries with quality steel products of various sizes. Each year, the main producers of rolled products in Kazakhstan (joint-stock company "Arselor Mittal Temirtau") and Russia (Magnitogorsk metallurgical combine, Novolipetsk metallurgical plant, Vyksa metallurgical plant, Cherepovets metallurgical plant) are upgrading their production to increase the volume of production and improve the quality of the manufactured metal products. The manufacturers of steel products in Russia and Kazakhstan (which are included in the Eurasian economic Union) have also another important task - to increase the competitiveness of the domestic sheet products in foreign markets. One of the ways to improve the quality of sheet metal and, accordingly, increase its competitiveness is to obtain strips and sheets with specified geometric dimensions.

The improvement of the technology of sheet rolling, which allows to obtain strips and sheets of specified geometrical dimensions while saving their flat shape, is directly related to the acquisition of new knowledge about the correlation between rolling parameters, which consequently leads to the necessity of development of mathematical models adequately reflecting the real processes in the hearth of plastic deformation during rolling. This work is aimed at development of a scientific basis for the formation of the transverse profile and flatness of thin strips, and in particular, at the development of a mathematical model of the process of broadening of rolled strips, which will allow to investigate the distribution of the broadening along the zone of plastic deformation, depending on the parameters of rolling and strip.

THEORETICAL STUDIES

The principle of minimum of total energy of deformation, based on the beginning of possible changes in the strain state of a deformable body (beginning of Lagrange), is widely used for the study of processes of metal forming.

A variation of the principle of possible changes in the strain state of deformable bodies is the variational principle of Jourdain, in accordance with which the variation is subject to the rate of metal flow in the plastic deformation zone [1 - 14].

The variational equation of Jourdain for plastic deformation zone is written as follows:

$$\delta\Big(\iiint\limits_{\Omega} \Pi_v \mathrm{H} d\Omega - \iint\limits_{S} \vec{\sigma}^n \vec{v} ds + \sum_{i=1}^{n} \iint\limits_{S_i} \tau_s \left|\Delta v_i\right| ds\Big) = 0 \quad (1)$$

where:

$\Pi_v = \int_0^{\mathrm{H}} \mathrm{T} d\mathrm{H}$ is speed potential;

T and H - the intensity of tangential stresses and velocities of shear deformation;

$\vec{\sigma}^n$, $\vec{v}$ - full stress on the surface S with outer normal $\vec{n}$ and corresponding speed of movement;

Δv_i - jump of velocities on i - th surface of shear S_i;

τ_s - yield strength of shear;

δ - varying symbol.

For rigid-plastic area equation (1) is written as follows:

$$\delta\Big(\iiint\limits_{\Omega} \tau_s \mathrm{H} d\Omega - \iint\limits_{S} \vec{\sigma}^n \vec{v} ds + \sum_{i=1}^{n} \iint\limits_{S_i} \tau_s \left|\Delta v_i\right| ds\Big) = 0 \quad (2)$$

The first integral represents the power of internal resistance; the second integral is the power of external forces at the boundaries of the deformation zone; the third integral is the power of shear forces.

Under the sign of variation in the equations (1) and (2) is the functionality, which is a full rolling power. Expressions for the extremals, where the functional takes the minimum value, can be found with the help of the Euler-Lagrange equations. In the case of using the variational principle of Jourdain, the desired extremals are functions describing the velocity field of metal flow in the zone of plastic deformation. To find analytical expressions of extremals is possible only in some simple cases, therefore, most expressions for the extremals are determined on a certain class of functions that satisfy boundary values of the assigned task. The method of determining the extremals of these is the Ritz method.

Using the Ritz method the variational equation of Jourdain for the case of rolling with tension in expanded form, is written as follows [2 - 9]:

$$\frac{\partial}{\partial a_j}\left(N_1 + N_2 + N_3 - N_4 + N_5\right) = 0 \quad (3)$$

where N_1 - power of internal resistance;

N_2 - power of the forces of sliding friction;

N_3 - power of shear forces;

N_4 - power of front tension;

N_5 - power of back tension;

a_j - varied parameters in the equations of velocities of the metal flow;

m - number of unknown parameters.

Under the sign of differentiation is the equation for the full power of rolling.

In Fig. 1 is showed the calculation scheme used to describe the process of broadening in the zone of plastic deformation.

The zone of plastic deformation consists of two areas – the zone of advance and the zone of the lag. The edge shape shown by the dashed line representing a smooth curve, for ease of computations is approximated to the area of the lag and lead lengths of two straight lines. The diagram uses the following notation:

v_0, v_1, v_x, v_y - input and output speeds of the strip and the projection of the velocity of the metal side edge on the axis x and y accordingly;

$h_0, h_n, h_1, B_0, B_n, B_1$ - thickness and width of strip at the entrance, in the neutral section and at the output accordingly;

ℓ, x_i - the length of the deformation zone and the advance zone.

In accordance with the adopted scheme, the equations describing the shape of the side edges of the strip in the zone of plastic deformation are written as follows:

a) advance zone $(0 \le x \le x_i)$;

$$B_{adv}(x) = B_0\left[\ 1 + \beta + (\beta_t - \beta)\frac{x}{x_i}\ \right]$$

b) lag zone $(x_i \le x \le \ell)$

$$B_{lag}(x) = B_0\left[\ 1 + \frac{\beta_t}{1-t_i} - \frac{\beta_t}{1-t_i}\frac{x}{\ell}\ \right]$$

where

$$\beta = \frac{\Delta B}{B_0},\quad \beta_t = \frac{\Delta B_t}{B_0},\quad \Delta B = B_1 - B_0$$

$$\Delta B_t = B_n - B_0,\quad t_i = \frac{x_i}{\ell}.$$

From considerations of the kinematic admissibility conditions, we get the following conditions for side edge:

a) advance zone $(0 \le x \le x_i)$;

$$\left.\frac{v_y}{v_x}\right|_{kp} = \frac{\Delta B - \Delta B_t}{x_i}$$

b) lag zone $(x_i \le x \le \ell)$

$$\left.\frac{v_y}{v_x}\right|_{kp} = \frac{\Delta B_t}{\ell - x_i}$$

For the speed of the transverse displacement of an arbitrary material point with the current coordinate in the zone of plastic deformation was appointed a power dependence in accordance with [2-7], which can show a different character increase of the speed of transverse displacement of metal from the middle of the strip to the edges:

a) advance zone $0 \le x \le x_i$,

$$\frac{v_y}{v_x} = \frac{\Delta B - \Delta B_t}{x_i}\left[\frac{y}{B(x)}\right]^p \tag{4}$$

b) lag zone $x_i \le x \le \ell$

$$\frac{v_y}{v_x} = \frac{\Delta B_t}{\ell - x}\left[\frac{y}{B_{lag}(x)}\right]^p$$

where p is a varied parameter.

The speed of longitudinal movement of the metal in the hearth of plastic deformation v_x is determined from the law of constancy of second volumes:

$$v_0 h_0 B_0 = v_1 h_1 B_1 = v_x h_x B_x = v_r h_i B_i \cos\gamma \tag{5}$$

where v_r is speed of the work roll;

γ - neutral angle;

R - radius of the work roll;

$\cos\gamma = \sqrt{1 - \left(\frac{x_i}{R}\right)^2}$ - cosine of the neutral angle;

$$h_x = h_1 + \Delta h\left(\frac{x}{\ell}\right)^2;\ h_i = h_1 + \Delta h\left(\frac{x_i}{\ell}\right)^2$$

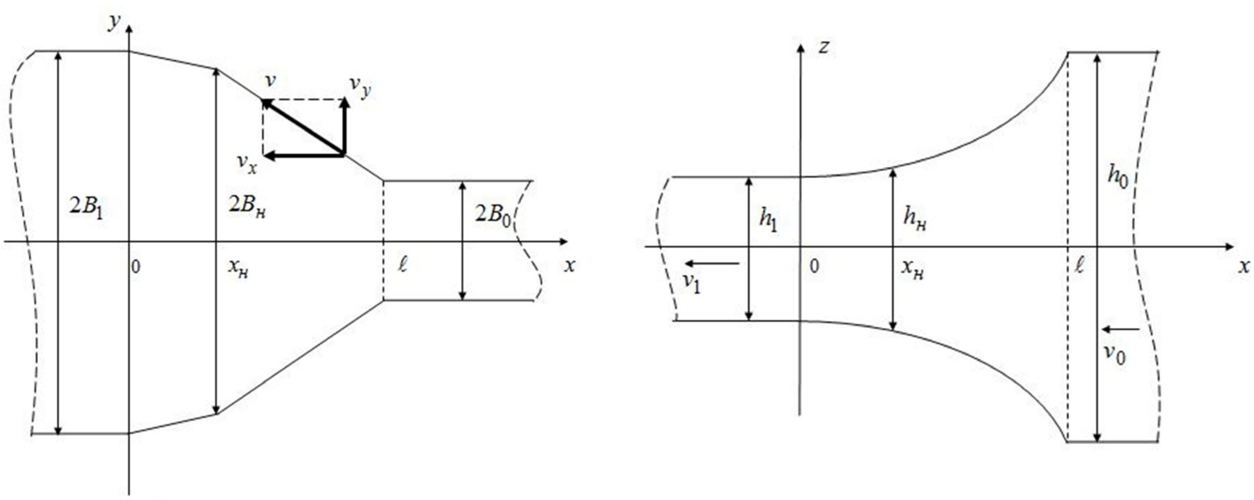

Fig. 1. Calculation scheme.

RESULTS AND DISCUSSION

Using known relations of the theory of plasticity and acting in the same way as in [2 - 3], expressions for the velocity and strain rates were found:

$$v_x = -\frac{A}{\left(1+\beta-\beta\frac{x}{\ell}\right)\cdot\left[a+\varepsilon\left(\frac{x}{\ell}\right)^2\right]} \quad (6)$$

where

$$A = \cos\gamma\cdot\left(1+\beta-\beta\frac{x_i}{\ell}\right)\cdot\left[a+\varepsilon\left(\frac{x_i}{\ell}\right)^2\right]$$

$$a = \frac{h_1}{h_0};\ \ \varepsilon = \frac{\Delta h}{h_0}.$$

The speed of transverse movement of the metal in the deformation zone:

a) advance zone $0 \le x \le x_i$,

$$v_y = v_x \frac{\Delta B - \Delta B_t}{x_i}\left[\frac{y}{B(x)}\right]^p \quad (7)$$

b) lag zone $x_i \le x \le \ell$

$$v_y = v_x \frac{\Delta B_t}{\ell - x_i}\left[\frac{y}{B(x)}\right]^p$$

The strain rate of the metal in the deformation zone:

$$\xi_x = \frac{\partial v_x}{\partial x},$$

$$\xi_y = \frac{\partial v_y}{\partial y},$$

$$\xi_z = -\xi_x - \xi_y,$$

$$\eta_{xy} = \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right),$$

$$\eta_{xz} = \frac{1}{2}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right),$$

$$\eta_{yz} = \frac{1}{2}\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right).$$

Taking that the deformation of the metal height is uniform, an equation for the speed of vertical movement of the metal was obtained:

$$v_z = \int \xi_z dz = -\left(\xi_x + \xi_y\right)\cdot z + \varphi(x, y) \quad (8)$$

From the condition of the symmetry of the problem, $v_z\big|_{z=0} = 0$. Therefore, $\varphi(x, y) \equiv 0$, and the speed of vertical movement of the metal in the zone of plastic deformation is:

$$v_z = -\left(\xi_x + \xi_y\right)\cdot z \quad (9)$$

The intensity of strain rates, based on the velocities of shear deformations, was calculated:

$$\mathrm{H} = \sqrt{\frac{2}{3}}\sqrt{\left(\xi_x - \xi_y\right)^2 + \left(\xi_y - \xi_z\right)^2 + \left(\xi_z - \xi_x\right)^2 + \frac{3}{2}\left(\eta_{xy}^2 + \eta_{yz}^2 + \eta_{zx}^2\right)} \quad (10)$$

The slip power between rolls and strip is:

$$N_{ск} = 4\mu\tau_s \int_0^B dy \int_0^\ell \sqrt{\Delta v_x^2 + v_y^2 + v_z^2}\, dx \quad (11)$$

where $\Delta v_x = (v_x - v_r)$ - the speed of sliding of the metal relative to the roll; μ - the friction coefficient.

The jump of vertical and horizontal speeds of the metal flow is taken into account by introducing the shear power in the input section of the zone of plastic deformation; in the neutral and the output section there is only a jump of the speed of the transverse flow of the metal, taken into account by the introduction of shear power in these sections.

The power of the front and back tension is calculated as the multiplication of the full tension on the output and input speeds of longitudinal movement of the strip.

The result was obtained as a system of three equations:

$$\begin{cases} \frac{\partial}{\partial p}\left(N_1 + N_2 + N_3 - N_4 + N_5\right) = 0 \\ \frac{\partial}{\partial(\Delta B)}\left(N_1 + N_2 + N_3 - N_4 + N_5\right) = 0 \\ \frac{\partial}{\partial(\Delta B_t)}\left(N_1 + N_2 + N_3 - N_4 + N_5\right) = 0 \end{cases} \quad (12)$$

Table 1. Experimental conditions.

№ exp.	h_0, mm	$2B_0$, mm	h_1, mm	$2B_1$, mm	Δh, mm	$2\Delta B$, mm	ℓ, mm
1	10,1	30,1	8,15	31,4	1,95	1,3	15,0
2	10,0	15,4	7,8	16,8	2,2	1,4	16,0

The system of equations (12) represents the mathematical model of the broadening process of rolled strips, which can be used to study the distribution of broadening in the deformation zone, depending on different rolling parameters and strip, including the tension.

To validate the developed model an experiment was conducted on a laboratory rolling mill; the diameter of work rolls was 120 mm. The experimental conditions are shown in Table 1.

In Fig. 2 the thin line shows the results of experiments, while the thick line presents the results of theoretical calculation. It is seen that the calculation results agree well with the experimental data.

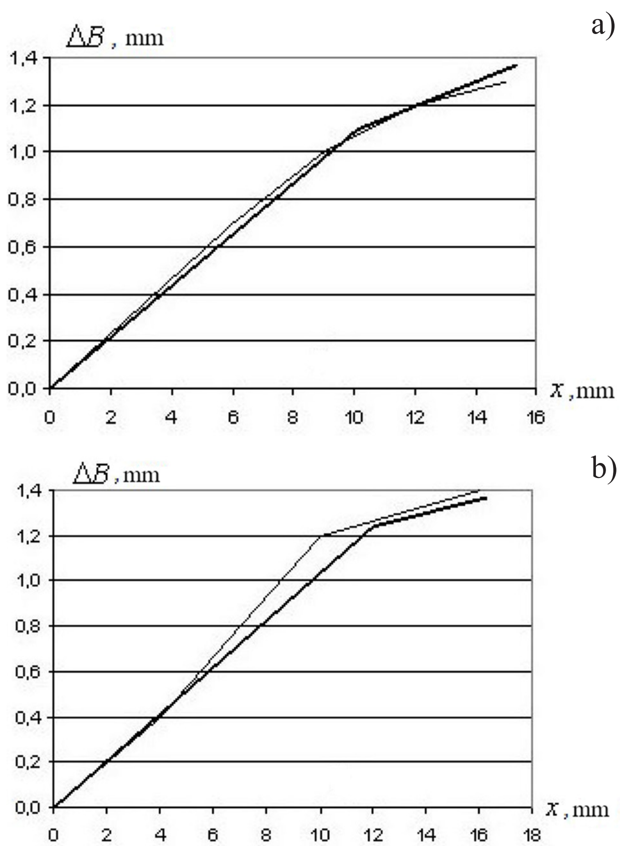

Fig. 2. The comparison of experimental and calculated data: a – 1st experiment; b – 2nd experiment.

CONCLUSIONS

The paper presents a mathematical model of the process of broadening during rolling of strips and sheets. A specific feature of the model is the consideration of metal flow in the two zones of plastic deformation hearth: advancing zone and lag zone. A mathematical model, based on the variational principle of minimum total capacity of rolling (the principle of Jourdain), has been developed. In accordance with this principle, the metal flow velocities in the plastic deformation hearth should be varying. Velocity of the metal transversal flow in the hearth of deformation is represented as a power function of a transversal coordinate. Three parameters are to be varied: value of total broadening, value of broadening in the lag zone and degree of the dependence of transversal metal flow velocity. These parameters are determined by the Ritz method. The comparison of experimental and calculated results demonstrated adequacy of the developed model.

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