METHOD OF CALCULATION OF STRAINS AND STRESSES ON THE WIDTH OF A THIN STRIP IN COLD ROLLING

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ABSTRACT

The method of calculation of unevenness of deformations and stresses on the width of a strip at sheet rolling is developed. It is shown that taking into account the elastic-plastic deformation, changes in the modulus of normal elasticity and the transverse flow of the metal during the formation of the strip can significantly improve the accuracy of determining the parameters characterizing the flatness of sheet metal. The experiments carried out during cold rolling of 1.0 mm thick and 250 mm wide strips of St.3 steel on a 200/350×500 four-roll mill confirmed the adequacy of the mathematical model when comparing the calculated and measured longitudinal stresses over the width of the thin strip.

Keywords: sheet rolling, flatness, forming, elastic-plastic deformation, transverse flow of metal, modulus of elasticity.

INTRODUCTION

Flatness is one of the most important indicators of the quality of sheet metal. Improvement of the flatness of sheet metal and production of strips, tapes and sheets without shape defects (“undulation”, „wave“ and „sickle“) provides a higher level of technological and consumer properties and quality of hot and cold rolled metal [1 - 4].

When analyzing the conditions for obtaining flat strips, it is assumed that a flat scheme of the deformation center prevails during cold rolling. It is believed that cross over the metal slightly, and therefore the constant extraction across the width of the strip, the sheet metal does not occur. However, with a slight unevenness of deformations $\delta\varepsilon$ and extracts $\delta\lambda$ ($\delta\mu$) along the width of the strip, defects "wave", "box" are formed (Fig.1) and an unevenness of longitudinal stresses $\delta\sigma$ is formed.

Formulation of the problem

The results of the experimental study [5] showed that when rolling strips under certain conditions, there is a significant discrepancy between the uneven longitudinal stresses along the width of the strip $\delta\sigma p$, determined analytically by the method based on the ideas of plane deformation, and the experimental determination of the same value $\delta\sigma e$. Typically, the calculated value is much larger than the measured value, i.e. $\delta\sigma_c >> \delta\sigma_e$. Currently, it is believed that in addition to the mechanism of self-leveling due to feedback, through the redistribution of elastic flattening of the roll under the strip, another mechanism acts to stabilize the rolling process – transverse deformation of the metal in the deformation center. The value of the transverse deformation of the metal is estimated by the coefficient ρ. Therefore, to assess the shape used the value of uneven longitudinal stresses across the width of the strip $\delta\sigma_c$, which is determined by the difference between the edge and the middle of the strip, as well as the value of the coefficient of transverse deformation of the metal.

A positive value $\delta\sigma_c$ denotes the tendency of the band to “wave”, a negative - to “box”. Taking into account the condition of non-planar deformation, the change in

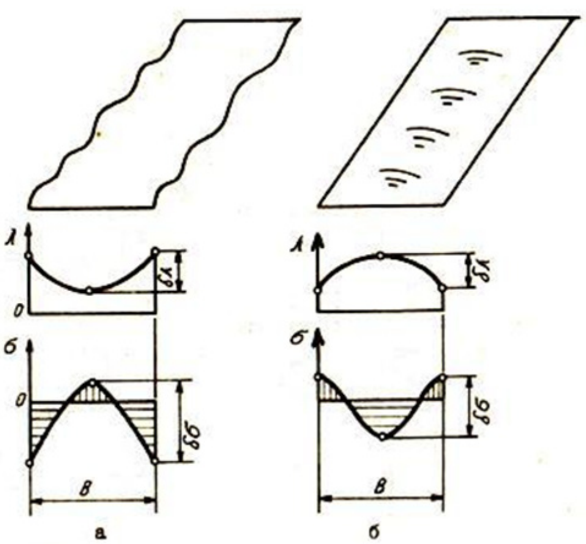

Fig. 1. Distribution of hoods and stresses over the width of the strip in the formation of undulation (a) and warpage (b).

the shape of the strip during rolling can be determined based on the characteristics of its profile before δh_{i-1} and after δh_i passage and the value of σ [5, 6].

$$\delta\sigma_c = \rho\left(\frac{\delta h_i}{h_i} - \frac{\delta h_{i-1}}{h_{i-1}}\right)E \tag{1}$$

where δh_{i-1} and h_{i-1} - transverse thickness (profile) and the thickness of the strip at the entrance to the i-th cage; δh_i and h_i - transverse thickness (profile) and the thickness of the strip at the exit of the i-th cage; E - modulus of elasticity of the metal; ρ - coefficient, taking into account the transverse deformation of the metal, $0 \le \rho \le 1$.

Experimental studies and calculations allowed us to establish that for the conditions of hot rolling in the last stand of the mill 2000 NLMK strips of steel Ст.3 thickness of 1.5 - 3.5 mm, width 1250 – 1400 mm, the value of ρ = 0.75 - 0.90. Experimental studies carried out on two laboratory-industrial mills: single-celled four-roll 205/360×500 and two-celled continuous four-roll 150/380×320 mm during cold rolling of strips of steel 08 kp thickness of 0.6 - 1.0 mm, width of 200 - 250 mm revealed a significant influence of uneven compression width of the strip on the coefficient ρ [5]. The analysis of the results of these studies allowed us to establish that with a relatively uniform compression over the width of the strip, when the shaping occurs in the region of elastic deformations, the coefficient ρ approaches one ($\rho \approx 1$), and decreases 20 - 25 times from 1.0 to 0.04 with an increase in the unevenness of deformation over the width of the strip from 0.2 to 2 %. It is known that when compressing more than 0.2 %, most metals and alloys change size and shape, i.e. deformed plastically. Therefore, when calculating the stress unevenness over the band width δσ in this area, the application of the elastic modulus E can lead to significant errors.

In work [6] for the first time analytical dependence for calculation of coefficient of transverse deformation of metal p on the basic parameters of process of sheet rolling has been received:

$$\rho = \frac{K^2}{K^2 + \pi^2} \tag{2}$$

In formula (2) the value of K^2 is calculated by the equation:

$$K^2 = 3 f_{\text{fr}} \tau_s h_0 B^2 / E\Delta h\, h_{\text{mid}} l \tag{3}$$

where f_{fr} - coefficient of a contact friction; τ_s - tangential (shear) stress; h_0 - the thickness of the strip at the entrance to the rolls; B - the width of the strip; E - the modulus of normal elasticity, for St.3 is $E = 21\times10^4$ N/mm^2; Δh - absolute compression; h_{mid} - the average thickness of the strip between the input and output of the rolls; l - the length of the deformation focus.

In [7], equation (3) is refined by introducing the parameter t_n – position of the neutral section in the deformation focus. The calculation found that rolling with high sliding speeds reduces by 30 % the coefficient of p from 0.42 to 0.32, which contributes to the production of flat strips. However, the parameter t_n depends on the advance, the accuracy of which, as a rule, is not very high.

EXPERIMENTAL

We investigate the accuracy of determining the irregularity of longitudinal stresses over the width of the band $\delta\sigma_c$ using equations (1 - 3) when compared with the results of measurements $\delta\sigma_e$ in the course of the experiments. The study was conducted on four-roll mill 200/350×500 (mill 500), the drive power of 200 kW, with rolling speed of 0.5 m/s. In the experiments was rolled with a strip tension of steel St.3 width 250 mm, thickness 1.0 mm. To create uneven deformation of the metal width used counter-bending of the work rolls. During rolling, the rolling force P, the anti-bending force Q of the working rolls, the back and front tension of the strip, the distribution of longitudinal stresses σ_e over the width of the strip at the exit of the mill and other indicators of the rolling process were recorded.

To control the distribution of longitudinal stresses across the width of the strip, a manufactured magnetoanisotropic sensor of the MISIS design was used [8]. During the rolling experiment, the longitudinal stress distribution was determined with an accuracy of ±1 N/mm^2 using a single sensor moved across the strip at a speed of about 4 m/s. To measure the profile strips was used profilometer with absolute accuracy of 1 micron. The value of the transverse thickness difference δh and the difference of longitudinal stresses $\delta\sigma_e$ between the middle and the edge of the strip (the first measurement was carried out at a distance of 20 mm from its edge) was considered positive if the thickness and longitudinal stresses in the strip decrease from the middle to the edge and vice versa. The final shape of the strip was judged by measuring the shape after cutting by amplitude and wavelength.

Rolling was carried out with rear σ_0 = 2,2 N/mm^2 and front σ_1 = 5,1 N/mm^2 tensions and with lubrication of the rolls with 6 % coolant based on mineral oil. According to the work [9] the coefficient of contact friction f_{fr} = 0.06. The unevenness of deformation and stresses $\delta\sigma_e$ along the width of the strip was carried out by the anti-bending of the working rolls with the force Q. The difference of compression δε and extracts δμ/μ along the width of the strip was determined by the equation

$$\delta\varepsilon = \delta\mu/\mu = \delta h_1 / h_1 - \delta h_0 / h_0 \qquad (4)$$

RESULTS AND DISCUSSION

The results of two experimental rolling mill 200/350×500 strips of lining thickness h_0 = 1.00 mm with a transverse thickness (convexity) δh_0 = 0.020 mm are shown in the Table 1.

Rolling of the strip with compression ε = 10 % in the mode No. 1 led to uneven compression along the width of the strip $\delta\varepsilon_1$ = 0.78 % and the formation of the measured difference of longitudinal stresses between the edge and the middle $\delta\sigma_e$ = 89 N/mm^2 with the formation of a defect of the form "undulation". The use of anti-bending of working rolls when rolling the strip according to the mode No. 2 allowed to reduce the unevenness of compression along the width of the strip to $\delta\varepsilon_2$ = 0.47 % and the difference in longitudinal stresses to $\delta\sigma_e$ = 37 N/mm^2, which led to a decrease in the parameters of the undulation of sheet metal.

Let us determine the unevenness (difference) of the longitudinal stresses $\delta\sigma_c$ along the width of the strip by calculation using equation (1) having previously calculated the cross-flow coefficient of the metal ρ. When rolling the strip according to mode No. 1 using equations (2) and (3) we obtain:

$$K^2 = 3 f_{fr} \tau_s h_0 B^2 / E\Delta h \cdot h_{mid} l =$$
$$= 3 \cdot 0,06 \cdot 160 \cdot 1,0 \cdot (250)^2 /$$
$$/ 210000 \cdot 0,1 \cdot 0,95 \cdot 3,46 = 26,08$$

Then the coefficient is equal to

$$\rho = \frac{K^2}{K^2 + \pi^2} = \frac{26,08}{26,08 + (3,14)^2} = 0,73$$

Let us calculate the coefficient ρ when rolling the strip according to mode No. 2:

$$K^2 = 3 f_{fr} \tau_s h_0 B^2 / E\Delta h \cdot h_{mid} l =$$
$$= 3 \cdot 0,06 \cdot 160 \cdot 1,0 \cdot (250)^2 /$$
$$/ 210000 \cdot 0,11 \cdot 0,94 \cdot 3,52 = 23,3$$

In this case the coefficient is equal to

$$\rho = \frac{K^2}{K^2 + \pi^2} = \frac{23,3}{23,3 + 9,86} = 0,70$$

The difference of longitudinal stresses over the width

Table 1. Influence of unevenness and deformation conditions on the difference of longitudinal stresses δσ in width during cold rolling of 1.0×250 mm strip of St.3 steel on a four-roll mill 200/350×500.

Mode	h_0, mm	h_1, mm	ε, %	P, kN	Q, kN	$\delta\sigma_c$, N/mm^2	δh_1, mm	ρ	l, mm	δε, %	$\delta\sigma_e$, N/mm^2	$\delta\sigma_{cc}$, N/mm^2
1	1.00	0.90	10	354	-	1196	0.025	0.73	3.46	0.78	89	83.1
2	1.00	0.89	11	381	21	691	0.022	0.70	3.52	0.47	37	30.9

of the strip σ_c in the assumption of elastic deformation of the metal according to equation (1) taking into account equation (4) when rolling the strip under the deformation mode No. 1 will be equal to:

$$\delta\sigma_{c1} = \rho\delta\varepsilon_1 E = 0{,}73 \cdot 0{,}0078 \cdot 210000 = 1195{,}7 \text{ N/mm}^2.$$

The calculated value of the difference in longitudinal stresses over the width of the strip $\delta\sigma_c$ = 1195.7 N/mm^2 is significantly 13 times greater than the value $\delta\sigma_e$ = 89 N/mm^2 measured in the rolling process (see Table 1).

Let us determine by equation (1) the difference in longitudinal stresses when rolling the strip under the deformation mode No. 2.

$$\delta\sigma_{c2} = \rho\delta\varepsilon_2 E = 0{,}70 \cdot 0{,}0047 \cdot 210000 = 690{,}9 \text{ N/mm}^2$$

Comparative analysis showed that the calculated value $\delta\sigma_{c2}$ = 690.9 N/mm^2 is also significantly 19 times greater than the value $\delta\sigma_e$ = 37 N/mm^2 measured in the rolling process (see Table 1).

A large error in the determination of $\delta\sigma_c$ by equation (1) is explained by the incorrect use of the elastic modulus E for deformations exceeding 0.2 %. The dependence (1) is valid under the condition that the difference in the relative deformation over the band width $\delta\varepsilon$ is in the elastic region, i.e. $\delta\varepsilon \leq 0.2$ %. In most cases of cold rolling, the distortion of the strip shape with the manifestation of non-planking in the form of a "wave" or "box" occurs in the elastic-plastic region, when $\delta\varepsilon > 0.2$ %, which is shown in our studies. Therefore, the method of calculating the unevenness of stresses across the width of the strip requires the use of a module characterizing the deformation conditions of the strip taking into account the hardening curve (diagram "stress-strain") of the rolled metal or alloy. The dependence of stress on strain on the hardening curve of this material, including rolled steel St.3, is characterized by two main areas. The area of elastic deformation, when $\varepsilon \leq 0{,}2$ %, at which the stress is constant and its value is equal to the modulus of normal elasticity E, for steel St.3 is E = 21×10^4 N/mm^2. In the second section, when $\varepsilon > 0.2$ %, the plastic deformation of the metal begins and the strip is in the elastic-plastic region, and its stress is characterized by the elastic modulus E and the tangent modulus EC. The tangent modulus $E_t = \delta\sigma/\delta\varepsilon$ is equal to the tangent angle of the tangent to the abscissa axis in the diagram of the dependence of the stress σ on the deformation ε at the point corresponding to the deformation value [10]. In our rolling modes, the bands $\delta\varepsilon_1$ = 0.78% and $\delta\varepsilon_2$ = 0.47%. The concept of "two modules" in deformation beyond elasticity was first proposed by F.S.Yasinsky, and further developed by T.Karman [10].

At deformation beyond the limits of elasticity T.Karman justified the need for the use of the resulting (reduced) module E_r and proposed a formula for its calculation in the deformation of metals having a rectangular cross-section

$$E_r = 4E \cdot E_t / \left(\sqrt{E_t} + \sqrt{E}\right)^2 \quad (5)$$

However, as a result of experimental studies carried out in [11, 12] it was found that the modulus of normal elasticity E changes with the increase in the degree of plastic deformation ε. This influence was not taken into account in the work [13], which led to overestimated results of the calculation of $\delta\sigma$. Moreover, when cold deformation of samples of steel St.3 on ε = 10 % modulus of normal elasticity in plastic deformation E_p is reduced by 12 - 18 % (average 15 %) compared with its value in the annealed state (E). This should be taken into account in the stress calculation equations for rolling strips, sheets and profiles. The equations that take into account the effect of plastic deformation on the elastic modulus of E_p when rolling low-carbon steel can be written as:

$$E_p = E(1 - m\varepsilon), \quad (6)$$

where E - the modulus of normal elasticity of the steel strip in the annealed state, N/mm^2; ε - the relative compression of the strip; m - the coefficient depending on the grade of steel, for St.3 m = 1,5.

Therefore, in equation (5), proposed by T.Karman, instead of the elastic modulus E, it is necessary to write the modulus E_p, taking into account the plastic deformation of the strip. Then the refined equation of T. Karman for calculation of the resulting modulus of E_r at rolling of a steel strip will take the form:

$$E_{rp} = 4E(1 - m\varepsilon) \cdot E_t / \left(\sqrt{E_t} + \sqrt{E(1 - m\varepsilon)}\right)^2 \quad (7)$$

In formulas (6, 7) for the elastic region, $E_t = E_{pr} = E_r = E$. In the elastic-plastic region corresponding to the conditions of strip rolling, the refined resultant module of E_{rp} takes some intermediate values between the elastic modulus at plastic deformation of E_p and the tangent modulus of E_t. The use of the T. Karman modulus in equation (1) to calculate the non-uniformity of longitudinal stresses across the width of the strip should increase the accuracy of their determination to assess the formation of the strip during sheet rolling. We calculate the resulting and tangent modules when rolling a strip of steel St.3 for the above conditions of deformation of the metal. The value of the tangent modulus E_t can be determined by the tensile diagram of steel St.3, given in a number of works, including [9, 10, 14] using the dependence:

$$E_t = \frac{\Delta\sigma_s}{\Delta\varepsilon} \tag{8}$$

When rolling the strip according to the deformation mode No. 1, when $\Delta\varepsilon = \delta\varepsilon = \delta\varepsilon_1 = 0.78\ \% = 0.0078$, we obtain $E_{t1} = 0.5\times10^4$ N/mm^2. When rolling the strip according to the deformation mode No. 2, when the unevenness of deformation across the width decreases and is $\Delta\varepsilon = \delta\varepsilon = \delta\varepsilon_2 = 0.47\ \% = 0.0047$, the tangent modulus is $E_{t2} = 0.3\times10^4$ N/mm^2.

Let us calculate by equation (7) the refined resultant module for rolling mode No.1:

$$E_{rp1} = \frac{4E(1-m\varepsilon_1)E_{t1}}{\left(\sqrt{Et_{t1}}+\sqrt{E(1-m\varepsilon_1}\right)^2} =$$

$$= \frac{4\cdot21\cdot10^4(1-1.5\cdot0.10)\cdot0.5\cdot10^4}{\left(\sqrt{0.5\cdot10^4}+\sqrt{21\cdot10^4(1-1.5\cdot0.10}\right)^2} =$$

$$= 1.46\cdot10^4\ \text{N/mm}^2$$

Then we define the refined resultant module E_{cp2} for rolling mode No. 2:

$$E_{rp2} = \frac{4E(1-m\varepsilon_2)E_{t2}}{\left(\sqrt{E_{t2}}+\sqrt{E(1-m\varepsilon_2}\right)^2} =$$

$$= \frac{4\cdot21\cdot10^4(1-1.5\cdot0.11)\cdot0.3\cdot10^4}{\left(\sqrt{0.3\cdot10^4}+\sqrt{21\cdot10^4(1-1.5\cdot0.11}\right)^2} =$$

$$= 0.94\cdot10^4\ \text{N/mm}^2$$

Using the dependence structure (1), we write an equation for calculating the non-uniformity of longitudinal stresses over the width of the strip when replacing the elastic modulus E with the refined resulting modulus E_r

$$\delta\sigma_{cc} = \rho\left(\frac{\delta h_i}{h_i}-\frac{\delta h_{i-1}}{h_{i-1}}\right)E_{rp}, \tag{9}$$

where

$$\delta\varepsilon_i = \left(\frac{\delta h_i}{h_i}-\frac{\delta h_{i-1}}{h_{i-1}}\right).$$

Let us define $\delta\sigma_{cc}$ when rolling the strip according to the mode No. 1 on the mill 200/350×500, when $\delta\varepsilon_1 = 0.0078$ (0.78 %):

$$\delta\sigma_{cc1} = \rho_1\delta\varepsilon_1E_{rp1} = 0.73\cdot0.0078\cdot14600 = 83.13\ \text{N/mm}^2.$$

We calculate by equation (9) $\delta\sigma_{cc}$ when rolling the strip on the mode number 2 on the mill 200/350×500, when $\delta\varepsilon_2 = 0.0047$ (0.47 %):

$$\delta\sigma_{cc2} = \rho_2\delta\varepsilon_2E_{rp2} = 0.70\cdot0.0047\cdot94000 = 30.93\ \text{N/mm}^2.$$

Comparison of results of calculation of stresses $\delta\sigma_{cc}$ by equation (9) with the measured values $\delta\sigma_e$ given in the Table 1 shows that the error does not exceed 17 %. Therefore, the analysis of morphogenesis by the width of the strip sheet mill can be recommended to use the method of calculation of uneven longitudinal stresses on the basis of equation (9), taking into account elasto-plastic deformation of metal and change of elastic modulus during plastic deformation.

CONCLUSIONS

• The method of calculation of longitudinal deformations and stresses along the width of the strip is developed, taking into account the transverse and elastic-plastic deformation of the metal, as well as the change in the value of the elastic modulus during sheet rolling. It is established that taking into account the conditions of deformation of the metal significantly affects the accuracy of forecasting the distribution of longitudinal stresses across the width of the strip.

• Based on the results of experimental and theoretical studies of the quantitative impact of non-uniformity of strain $\delta\varepsilon$, coefficient of lateral flow of metal ρ and refinement method for the determination of the result module of E_{rp} characterizing elastic-plastic deformation on the formation of longitudinal stresses across the width of the strip. It is shown that when cold rolling strips with a cross section of 1.0×250 mm of steel St.3 on a four-roll mill 200/350×500 with a decrease in the compression difference in width $\delta\varepsilon$ from 0.78 to 0.47 % decreases the value of the resulting module E_{rp} from 1.46×10^4 to 0.94×10^4 N/mm^2, which leads to a decrease in the unevenness of the longitudinal stresses calculated with an accuracy of 17 %, the width of the strip $\delta\sigma_{cc}$ from 83.13 N/mm^2 to 30.93 N/mm^2 and improve its quality on the tablet.

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