EXPONENTIAL MOVING AVERAGE FOR AIR POLLUTION DATA: ASSESSING ITS ROLE IN PM_{10} MONITORING ACCURACY

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ABSTRACT

Accurate estimation of particulate matter (PM_{10}) concentrations is critical for assessing air quality and mitigating public health risks. Traditional monitoring data processing methods, such as simple moving averages (MA), often struggle to capture rapid fluctuations in pollutant levels due to their uniform weighting of historical data, potentially compromising real - time decision - making. This study evaluates the efficiency of the Exponential Moving Average (EMA) algorithm, which prioritizes recent observations through exponential weighting, to improve PM_{10} concentration estimates. Using data from urban air quality monitoring stations, EMA was applied across varying time windows and compared against conventional MA approaches. Performance was assessed against ground - truth measurements. Results demonstrated that EMA significantly reduced estimation errors. The algorithm exhibited enhanced responsiveness to abrupt PM_{10} spikes, attributed to its dynamic weighting mechanism. Sensitivity analysis revealed that optimal smoothing factors depended on the selected time window, balancing noise reduction and trend detection. These findings underscore EMA's potential as a robust tool for air pollution monitoring data analyses, offering superior adaptability to temporal variability. Implementation of EMA in regulatory and public health frameworks could enhance early warning systems and pollution control strategies. Future research should explore integrating EMA with machine learning models and low - cost sensor networks to further optimize real - time air quality management.

Keywords: PM_{10}, exponential moving average, air pollution monitoring, time - series analysis, air quality management.

INTRODUCTION

Air is the most mobile of the major environmental components and therefore plays an important role in the transport of various pollutants from and to other components. Identifying the changes that have occurred in air quality, the underlying causes, and predicting its future condition is one of the most important tasks of ecology for the protection of clean ambient air.

Air pollutants

Air pollutants can either be composed of solid matter or gaseous particles. While certain air pollutants are monitored more commonly than others, it is important to recognize that air pollution is a dynamic phenomenon, and any measurement we derive provides only a snapshot for one pollutant at a given point in time.

Creating denser, higher - resolution monitoring networks for proxy pollutants such as particulate matter and nitrogen dioxide helps us estimate the overall level of pollution in the air. The solid particles which form particulate matter (PM) vary in size, ranging from ultrafine to coarse particles. PM also varies in composition, making it important to measure the relative contribution of especially toxic PM such as black carbon.

Indoor, local and regional air quality are highly dynamic, complex systems that can change rapidly based on interactions between different pollutants, wind, and other climatic factors.

Dust is a major air pollutant. Its harmful health effects depend mainly on the size and chemical composition of the suspended dust particles, the other chemical compounds adsorbed on their surface, including mutagens, DNA - modulators, etc., and the compartment of the respiratory system in which they are deposited. The current definition comes from the National Air Quality Standard (NAAQS) on particulate matter (PM) introduced in 1987 by the US Environmental Protection Agency (EPA) [1]. Atmospheric particulate matter is defined as any suspended substance, solid or liquid, in which the individual aggregates are larger than the individual small molecules (about 0.0002 μm in diameter) but are smaller than about 500 μm. There are various methods for measuring fine particulate matter. Careful selection of the fine particle concentration measurement strategy is particularly important, as the value can vary widely depending on the time and location of the measurement as well as the method used.

Particulate matter (PM) is the most monitored air pollutant, with thousands of global measurement points reported by air quality databases. This number is growing rapidly with the advent of more reliable, affordable, and scalable air quality sensors.

While measuring PM at high - resolution allows us to answer the question of “is air pollution present?”, introducing additional instrumentation and methods allows us to answer a much wider range of questions such as:

- What specific sources are causing this air pollution?
- How do wind, photochemical reactions, and other climatic factors influence the development of air pollution in this area?
- How do different air pollutants contribute to health impacts?
- How can we most effectively minimize the negative impacts of air pollution?

Methods for measuring of PM_{10}

The current state of the ambient air is assessed by the concentrations of various pollutants in it. Data on the current state of the air can be collected using fixed and mobile stations implementing different technical approaches to measure the concentration of pollutants.

In Bulgaria, two methods are mainly used to measure fine particulate matter - gravimetric (which is the reference method) and the beta absorption method (which is used in AMS automatic measuring stations for ambient air quality monitoring).

In addition to these, the use of the Laser Light Scattering method in public distributed sensor networks (AirTube and similar) has gained popularity in recent years, mainly due to its low cost and ease of technical implementation [2].

Reference methods

The most used reference methods for measuring the content of fine particulate matter in the air are presented below.

- FBMS - Filter Based Manual Sampling
- BAM - Beta Attenuation Monitor
- TEOM - Tampered Element Oscillating Microbalance
- CI - Cascade Impactor
- DB - Diffusion Battery

Estimation methods

Mass of particles per volume is estimated by internal model. The most common and frequently used due to their simple technical implementation are the following:

- LLS - Laser Light Scattering
- APS - Aerodynamic Particle Sizer
- SMPS - Scanning Mobility Particle Sizer

The measurement method used in all automatic PM_{10} concentration monitoring stations in the city of Sofia is BAM method [3]. Measuring stations for PM_{10} locations in Sofia are shown on Fig. 1.

Public monitoring network sensors’ locations are shown on Fig. 2.

Public monitoring network sensors use LLS estimation method with internal model for determining particulate matter air concentration. [5]

In terms of measurement data collection, we could distinguish three main approaches in the methods used - low - speed (gravimetric method), medium - speed (beta - absorption) and high - speed (LLS sensors, one measurement every few s or min). Obviously, LLS has tens of thousands of times higher possible measurement frequency, and this brings both advantages

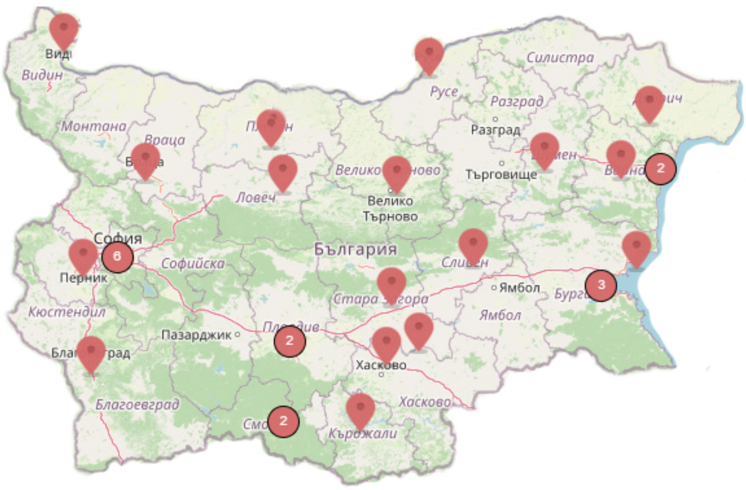

Fig. 1. Official government AMS using reference BAM method [4].

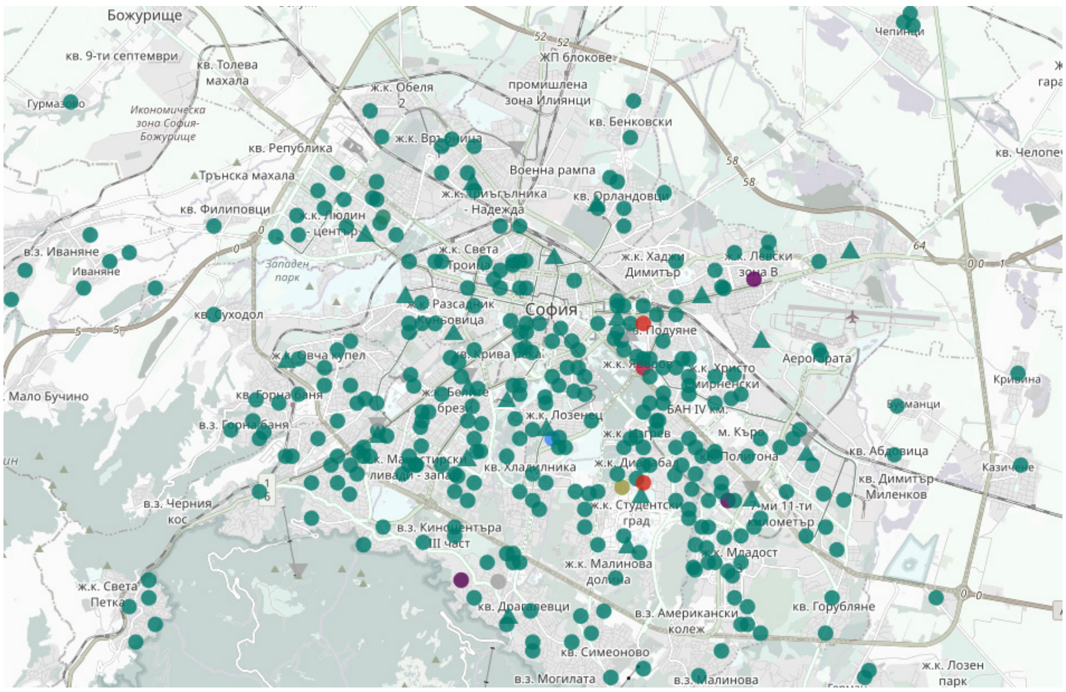

Fig. 2. Public (Luftdaten project) monitoring network using LLS estimation method with internal model [6].

and disadvantages when using the method.

Although high - frequency data collection has significant advantages for identifying burst emission events, it is unsustainable with respect to single significant deviations and measurement errors that could be due to a vast variety of factors.

Raw sample data filtering

Particularly useful in this regard are various methods for smoothing and filtering the high - frequency data coming from such measurement sensors.

To these we could include Exponential Weighted Moving Average (EWMA) and its variants. Also

Table 1. Relative data sampling frequency.

Type	Method		Minimal sampling frequency	Relative sampling frequency		Data collection frequency
reference	FBMS	gravimetric	8 h	1	0.03	24 h
	BAM	indirect mass estimation after fraction separation	15 min	32	1	1 h
estimation	LLS	model based	1 sec	28800	900	1 min

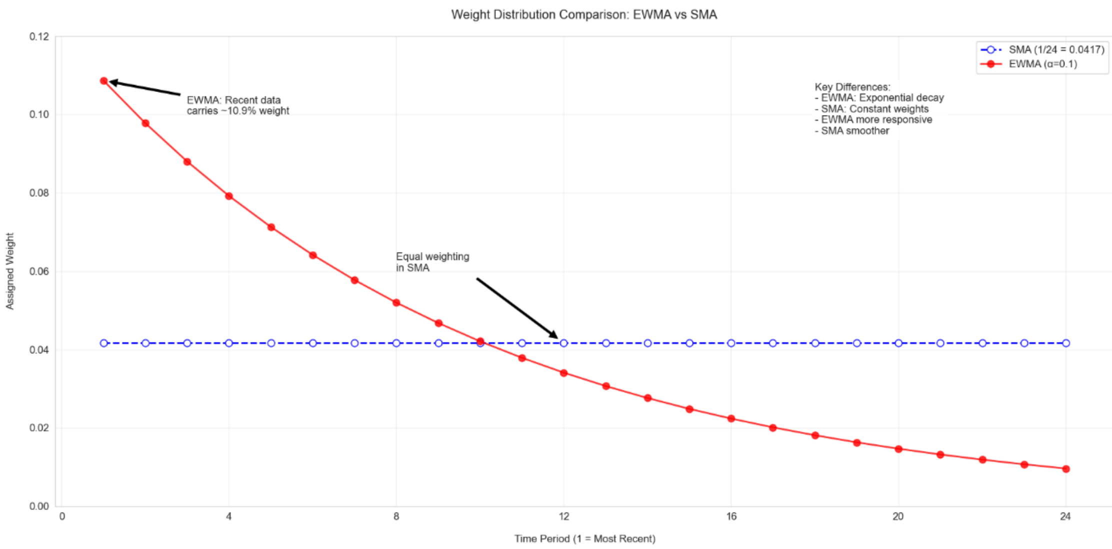

Fig. 3. Simple and exponentially smoothed moving average of a series of 24 measurements.

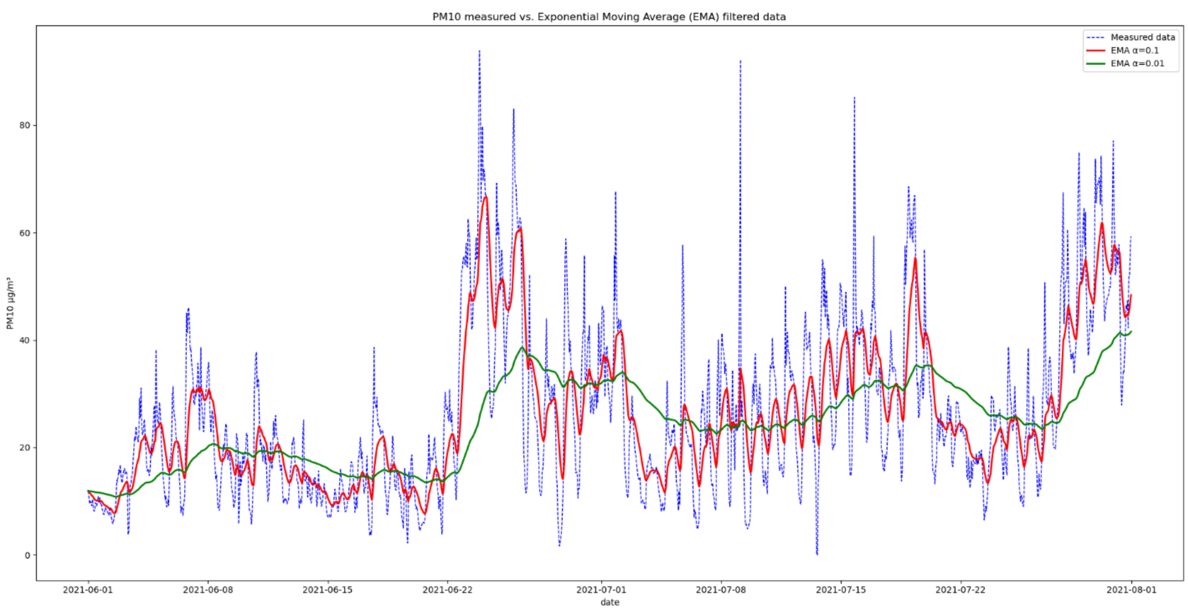

Fig. 4. Application of the EWMA algorithm to sensor data from PM_{10} $\mu g \cdot m^{-3}$ monitoring in the AMS “Kamentza” - Plovdiv for a two - week period.

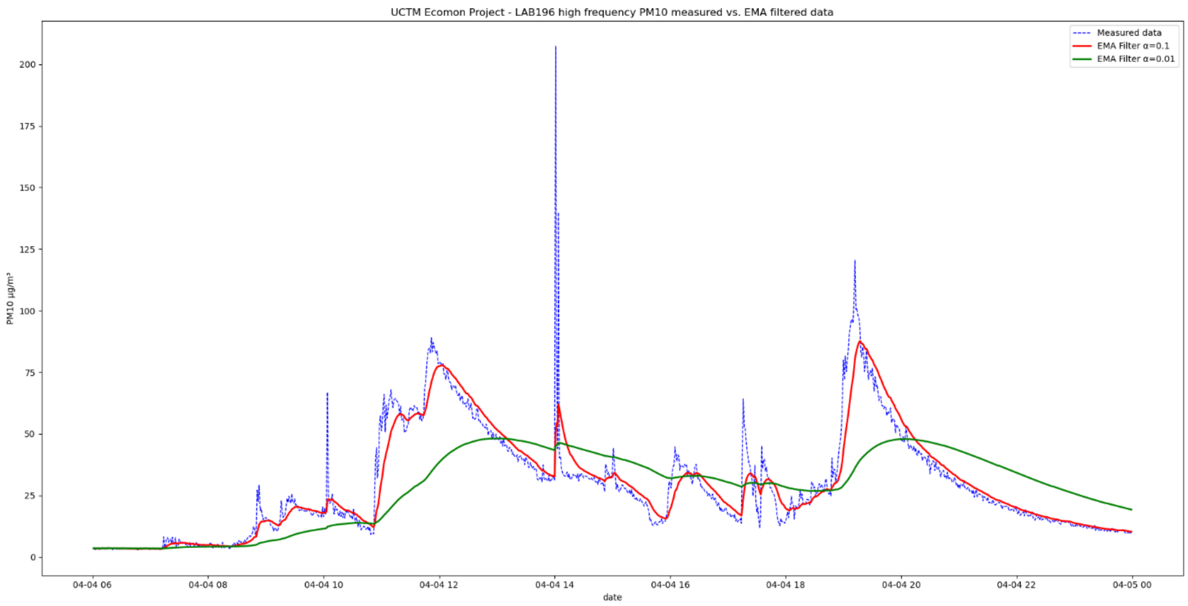

Fig. 5. Application of the EWMA algorithm to high - frequency sensor data from classroom dust monitoring PM_{10} μg m^{-3}.

particularly useful are the various bandpass filters that would help to obtain measurements close to those realized by the slow reference methods - gravimetric analysis and beta absorption.

EXPERIMENTAL

Exponentially weighted moving average

The Exponentially Weighted Moving Average (EWMA) is a quantitative or statistical measure used to model or describe a time series. The moving average is designed as such that older observations are given lower weights. The weights fall exponentially as the data point gets older (Eq. 1).

$$EWMA_t = \alpha \times r_1 + (1 - 2) \times EWMA_{t-1} \qquad (1)$$

The EWMA is a recursive function - the current observation is calculated using the previous observation.

The EWMA's recursive property leads to the exponentially decaying weights of older measurements and bigger importance in average calculation of recent data.

Fig. 3 shows the weight that the EWMA algorithm assigns to the input data when averaging a window of 24 h measurements, with the newest data at the beginning of the abscissa and the weighting coefficient on the ordinate. It should be also mentioned that EWMA's weights are front - loaded, emphasizing recent data more, whereas Simple Moving Average (SMA) gives equal importance to all data in the window. This makes EWMA more responsive to recent changes, while SMA is smoother but lagging.

Fig. 4 and Fig. 5 show applications of the EWMA algorithm in smoothing low - frequency (hourly average) and high - frequency (15 sec) data from sensor networks for monitoring air quality in an urban environment (Fig. 4, AMS "Kamenitza" - Plovdiv) and in a classroom (Fig. 5, "Ecomon" Project, UCTM Sofia).

Python program code for EWMA evaluation

A Python script was created for plotting the data:

```
alpha1 = 0.1
alpha2 = 0.01

# Initialize the EMA to be the same as the first data point
ema1 = df2.pm10[0]
ema2 = df2.pm10[0]

# Compute the EMA for each subsequent data point
ema1_values = [ema1]
ema2_values = [ema2]

for i in range(1, len(df2.pm10)):
    ema1 = alpha1 * df2.pm10[i] + (1 - alpha1) * ema1
```

```
        ema1_values.append(ema1)
       ema2 = alpha2 * df2.pm10[i] + (1 - alpha2) * ema2
        ema2_values.append(ema2)


    df2['pm10_EMA1'] = ema1_values
    df2['pm10_EMA2'] = ema2_values

    # plot predictions
    plt.figure("Figure No 4")
    plt.plot(df2.pm10, color='b', label='Measured
data', linestyle='dashed', linewidth='1')
    plt.plot(df2.pm10_EMA1, color='r', label='EMA
Filter α=0.1', linestyle='solid', linewidth='2')
    plt.plot(df2.pm10_EMA2, color='g', label='EMA
Filter α=0.01', linestyle='solid', linewidth='2')
    plt.title('UCTM Ecomon Project - LAB196 high
frequency PM10 measured vs. EMA filtered data')
    plt.xlabel('date')
    plt.ylabel('PM10 µg·m-3')
    plt.legend()
    plt.show()
```

The program implementation for an ATMega328p microcontroller uses the following code:

```
    int sensorPin = 0;      // input pin for ADC conversion
    int sensorValue = 0;            // Initialize initial
value from sensor EMA Y
    float EMA_a = 0.1;              // Initialize EMA
alpha
    int EMA_S = 0;                  // Initialize EMA S

    void setup(){
       Serial.begin(115200);
      EMA_S = analogRead(sensorPin); // Initial setup
of EMA S for t=1
    }

    void loop(){
      sensorValue = analogRead(sensorPin); // Current
value of the input variable
           EMA_S = (EMA_a*sensorValue) +
((1-EMA_a)*EMA_S);   // Calculate EMA
       Serial.println(EMA_S);                    // Print
EMA S
    }
```

The calculation of α can be done at the desired cut - off frequency of the low - pass filter, implemented by the EWMA (Eq. (2)).

$$y[n] = (1-\alpha).[n-1] + \alpha x[n] \quad (2)$$

where x[n] are the input data, and y[n] are the calculated values from the EMA. After applying the z - transform, we get Eqs. (3, 4):

$$Y(z) = (1-\alpha)z^{-1}Y(z) + \alpha X(z) \quad (3)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\alpha}{1-(1-\alpha)z^{-1}} \quad (4)$$

For normalized discrete radial frequency Ω [0, π], Eq. (5):

$$\left|H(z = e^{-i\Omega_{3dB}}\right|^2 = \frac{1}{2} = \left|\frac{\alpha}{1-(1-\alpha)e^{-i\Omega_{3dB}}}\right|^2 =$$

$$= \frac{\alpha^2}{|1-(1-\alpha)\cos(-\Omega_{3dB})-j(1-\alpha)\sin(-\Omega_{3dB})|^2} \quad (5)$$

From which the following Eqs. (6, 7) are obtained:

$$[1-(1-\alpha)\cos(\Omega_{3dB})]^2 + $$
$$+ [(1-\alpha)\sin(\Omega_{3dB})]^2 = 2\alpha^2 \quad (6)$$

$$\alpha^2 + 2(1-\cos(\Omega_{3dB}))\alpha - 2(1-\cos(\Omega_{3dB})) = 0 \quad (7)$$

With solutions from which only the positive solution can give a positive value of α (Eq. (8)).

$$\alpha = \cos(\Omega_{3dB}) - 1 \pm$$
$$\pm\sqrt{cos^2(\Omega_{3dB}) - 4\cos(\Omega_{3dB}) + 3} \quad (8)$$

Using this solution of the Eq. (8) for α and the following dependence, Eq. (9):

$$\Omega_{3dB} = \frac{\pi}{F_s/2} f_{3dB} \quad (9)$$

We can successfully determine α with given f_{3dB} and F_S

For calculating EWMA filter frequency response was developed.

```
    Import numpy as np
    import matplotlib.pyplot as plt
    from scipy.signal import freqz
```

```
# Parameters
Fs = 40000
f3db = 1

# Calculate omega3db
omega3db = f3db * np.pi / (Fs / 2)

# Calculate alpha
alpha = np.cos(omega3db) — 1 + np.sqrt(np.cos(omega3db)**2 — 4 * np.cos(omega3db) + 3)

# Filter coefficients
b = [alpha]
a = [1, -(1 — alpha)]

# Frequency response
W, H = freqz(b, a, worN=32768, fs=Fs)

# Plot the frequency response
plt.figure(1)
plt.plot(W, 20 * np.log10(np.abs(H)), label='Frequency Response')
plt.plot([f3db, f3db], [-40, 0], 'r—', label='3dB Point')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Magnitude Response (dB)')
plt.title('EWMA Filter Frequency Response')
plt.grid(True)
plt.legend()
plt.show()

# Frequency response near DC
W2 = np.arange(0, 76) * np.pi / (Fs / 2)  # 0 to 75 Hz
_, H2 = freqz(b, a, worN=W2)
W2 = W2 / (np.pi / (Fs / 2))  # Convert to Hz

# Plot the frequency response near DC
plt.figure(2)
plt.plot(W2, 20 * np.log10(np.abs(H2)), label='Frequency Response near DC')
plt.plot([f3db, f3db], [-20, 0], 'r—', label='3dB Point')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Magnitude Response (dB)')
plt.title('EWMA Filter Frequency Response near DC')
plt.grid(True)
plt.legend()
plt.show()
```

Fig. 6 shows EWMA frequency response.

Double exponential moving average (DEMA)

The data smoothing algorithm using a double exponentially weighted moving average was proposed in 1994 by Patrick G. Mulloy as an effective method for smoothing data while reducing the delay in reflecting rapid changes in the input data, Eq. (10) [7].

$$DEMA = 2 * EMA - EMA(EMA) \tag{10}$$

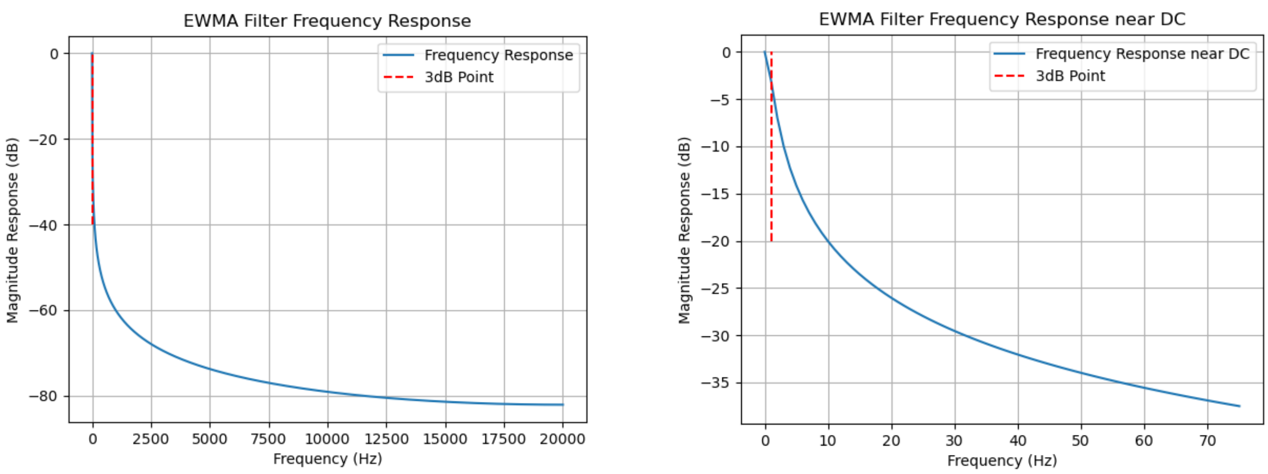

Fig. 6. EWMA Frequency response.

The algorithm requires 2*period - 1 data points to generate a smoothed value. The software implementation uses the following code:

```
int EMA_function(float alpha, int latest, int stored);
int ema_a = 0.06;
int ema_ema = 0;
int ema = 0;
int sensor_pin = 0;

void setup() {
}

void loop() {
  int sensor_value = analogRead(sensor_pin);

  ema = EMA_function(ema_a, sensor_value, ema);
  ema_ema = EMA_function(ema_a, ema, ema_ema);
  int DEMA = 2*ema – ema_ema;
}

int EMA_function(float alpha, int latest, int stored){
  return round(alpha*latest) + round((1-alpha)*stored);
}
```

Triple exponential moving average

The triple exponential weighted moving average algorithm was proposed again in 1994 by Patrick G. Mulloy. It builds on the idea behind DEMA (Double Exponential Moving Average) and works according to the Eq. (11):

$$TEMA = 3 * EMA - 3 * EMA(EMA) + EMA(EMA(EMA)) \tag{11}$$

The algorithm needs 3*period - 2 data points to generate a smoothed value.

Hull moving average (HMA)

The Hull Moving Average (HMA) attempts to minimize the lag of a traditional moving average while retaining the smoothness of the moving average line. It makes use of two separate weighted moving averages to prioritize more recent values and greatly reduce lag.

The following code was developed:

```
def weighted_moving_average(series: List[float], lookback: Optional[int] = None) -> float:
    if not lookback:
        lookback = len(series)
    if len(series) == 0:
        return 0
    assert 0 < lookback <= len(series)
    wma = 0
    lookback_offset = len(series) - lookback
    for index in range(lookback + lookback_offset - 1, lookback_offset - 1, -1):
        weight = index - lookback_offset + 1
        wma += series[index] * weight
    return wma / ((lookback ** 2 + lookback) / 2)
```

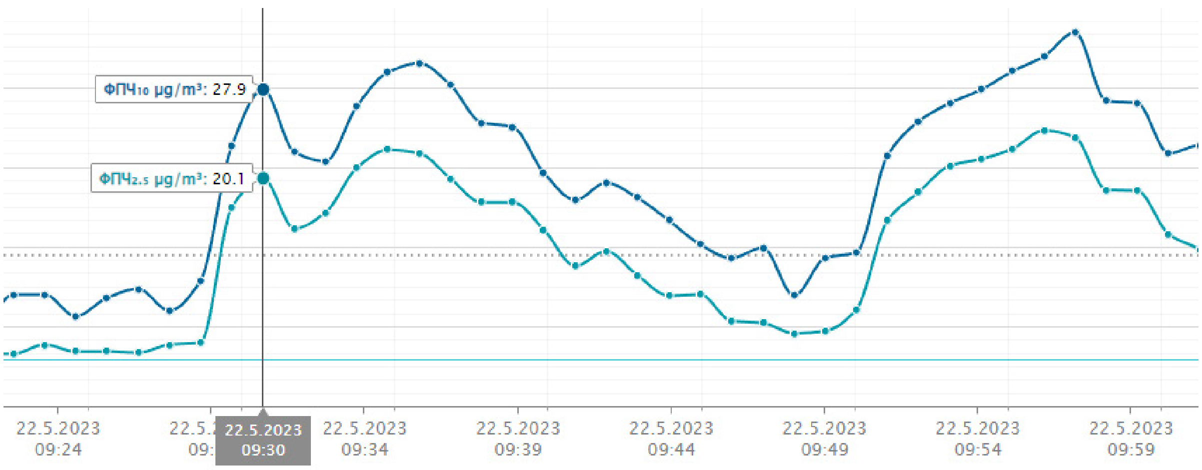

Fig. 7. HMA used for calculating 1 - min PM_{10} averages from high - frequency (1Hz) indoor sensor data.

```
def hull_moving_average(series: List[float], lookback: int) -> float:
    assert lookback > 0
    hma_series = []
    for k in range(int(lookback ** 0.5), -1, -1):
        s = series[:-k or None]
        wma_half = weighted_moving_average(s, min(lookback // 2, len(s)))
        wma_full = weighted_moving_average(s, min(lookback, len(s)))
        hma_series.append(wma_half * 2 - wma_full)
    return weighted_moving_average(hma_series)
```

RESULTS AND DISCUSSION

EWMA is an effective and easily applicable statistical method used to model or describe time series. The basic concept of EWMA is to give exponentially decreasing weights to older observations, thus making the algorithm more sensitive to recent changes in the data.

This provides several general advantages: As more recent observations gain more weight, EWMA responds more quickly to sudden changes or shifts in trends compared to other methods such as the simple moving average, which gives equal weight to all data points within a certain period. This sensitivity is especially useful when observing processes where early detection of deviations is important. EWMA can smooth out noise in time series. By assigning decreasing weights to older data, EWMA can help filter out random fluctuations and highlight key trends. It is a computationally efficient algorithm with low memory requirements, as only the previous EWMA value and the current monitoring are needed to calculate the current value. This efficiency makes it suitable for analysing large data sets and applications in real-time.

In the analysis of air pollution data, which is often characterized by high variability and autocorrelation, the EWMA algorithm offers several specific advantages:

Responsiveness to recent changes in pollution levels: Air pollution levels can change rapidly due to various factors such as traffic, industrial emissions, and weather conditions. EWMA, with its focus on recent data (controlled by the alpha parameter), allows timely detection of these changes. A higher alpha value will make EWMA more sensitive to recent fluctuations, which is important for quickly identifying peaks in contamination or other anomalies. This ability to quickly report on recent pollution events can be crucial for issuing timely health warnings or implementing short-term pollution control measures.

Efficiency when working with autocorrelated air quality data: Air quality data often shows autocorrelation - past values are related to current values. For example, a day with high pollution can be followed by another day with high pollution due to weather conditions. Although standard EWMA assumes data independence, research has shown that it can be effectively applied to autocorrelated data, especially after preprocessing with ARIMA models. This approach involves first modelling the autocorrelation in air quality data using ARIMA models and then applying EWMA to the residuals (the part of the data that is not explained by autocorrelation). This allows the observation of deviations from the expected autocorrelated model.

Application in air quality monitoring and control cards: EWMA is a key component in statistical process control (SPC) and is used to create control cards to monitor air quality parameters over time and to detect when a process gets out of control. Control maps provide a visual and statistical framework for continuously monitoring air quality and identifying significant deviations from the norm that may trigger investigations or interventions.

Sensitivity in detecting small changes and abnormalities in air pollutants: EWMA is known for its ability to detect small, permanent changes in the mean of the process more efficiently than other methods. This sensitivity is important in air quality monitoring to identify gradual increases in pollution levels that may not trigger alarms in less sensitive methods. The choice of the smoothness parameter and the width of the control boundaries affect the sensitivity of the control cards - smaller values result in a smoother map that is more sensitive to small, continuous changes, while larger values are more sensitive to sudden, large changes.

The experimental evaluation of particulate matter measurement methods was conducted in a controlled indoor environment, specifically a university study hall. Data were collected over an eight-day period using two distinct measurement techniques. A reference filter-based measurement system (FBMS) provided benchmark PM_{10} concentrations with a 24 h sampling

Table 2. Results from experimental evaluation of EWMA, parameter PM_{10}, μg m^{-3}.

Method	Filter	Sampling interval	N	N*	Average, μg m^{-3}	RMSE
FBMS	Raw Data	24 h	8	0	17.83	
LLS	Raw Data	15 sec	46080	701	24.83	15.3
LLS	EWMA, α = 0.1				19.36	13.9
LLS	EWMA, α = 0.01				15.45	14.2
LLS	DEMA				19.77	11.1
LLS	TEMA				19.96	11.2
LLS	HMA				18.05	8.1

interval. Concurrent measurements were performed using a light-scattering (LLS) method, capturing data at a higher frequency with a sampling interval of 15 sec. This parallel data acquisition strategy enabled a comparative analysis of the two methods' performance characteristics.

Table 2 presents the results of the experimental evaluation, focusing on the performance of the LLS method with various digital filtering techniques applied to the high-frequency data. The raw data from the LLS method exhibited a higher average PM_{10} concentration (24.83 μg m^{-3}) and root mean square error (RMSE = 15.3) compared to the FBMS reference (average = 17.83 μg m^{-3}). Application of an exponentially weighted moving average (EWMA) filter to the LLS data demonstrated a convergence of the mean towards the reference method's measurement. Specifically, EWMA with α = 0.01 yielded an average of 15.45 μg m^{-3}, closer to the FBMS value. Furthermore, the use of digital filters, including EWMA, Double EWMA (DEMA), Triple EWMA (TEMA) and HMA, resulted in a reduction in the RMSE, indicating improved accuracy in approximating the reference method's measurements. The HMA method demonstrated the lowest RMSE (8.1) among the filters applied.

The results indicate that applying exponential smoothing methods to high - frequency raw data from LLS sensors improves the agreement with slower reference measurement methods. The convergence of the mean and reduction in RMSE for daily averages suggest that these digital filtering techniques are effective in processing LLS data to provide more accurate estimates of PM_{10} concentrations. However, the optimal performance of these methods is contingent on careful tuning of the digital filter parameters. Subsequent investigations, conducted over extended periods and in both indoor and outdoor settings, have further validated the applicability of exponential smoothing methods and underscored the importance of parameter optimization for achieving convergence with reference measurements.

CONCLUSIONS

The study tests the application of smoothing methods on high - frequency data obtained by the LLS method, which is relatively inaccurate and unstable to errors in individual measurements. The EWMA variants demonstrate the ability to improve RMSE compared to measurements from reference methods after careful adjustment of the filter parameters. This enables the data from LLS sensors to be used with higher reliability when compared to the usual measurement intervals for PM_{10} concentration (hourly and daily average).

Authors' contributions: *The authors equally contributed to the conception, drafting, and final approval of the manuscript.*

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